

Predictive Computing for Solids and Liquids

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Schrödinger equation for a water molecule

10-particle, 30-dimensional partial differential equation

$$\hat{H} \left(-\frac{\hbar^2}{2m_e} \sum_{i=1}^{10} \nabla_i^2 - \frac{\hbar^2}{2m_I} \sum_{I=1}^3 \nabla_I^2 - \sum_{I=1}^3 \sum_{i=1}^{10} \frac{Z_I e^2}{4\pi\epsilon_0 r_{iI}} + \sum_{i=1}^9 \sum_{j=i+1}^{10} \frac{e^2}{4\pi\epsilon_0 r_{ij}} + \sum_{I=1}^2 \sum_{J=I+1}^3 \frac{Z_I Z_J e^2}{4\pi\epsilon_0 r_{IJ}} \right) \Psi = E\Psi$$

$$\frac{\partial^2}{\partial r_i^2} + \frac{2}{r_i} \frac{\partial}{\partial r_i} + \frac{1}{r_i^2} \left(\frac{1}{\sin^2 \theta_i} \frac{\partial^2}{\partial \phi_i^2} + \frac{1}{\sin \theta_i} \frac{\partial}{\partial \theta_i^2} \sin \theta_i \frac{\partial}{\partial \theta_i^2} \right)$$

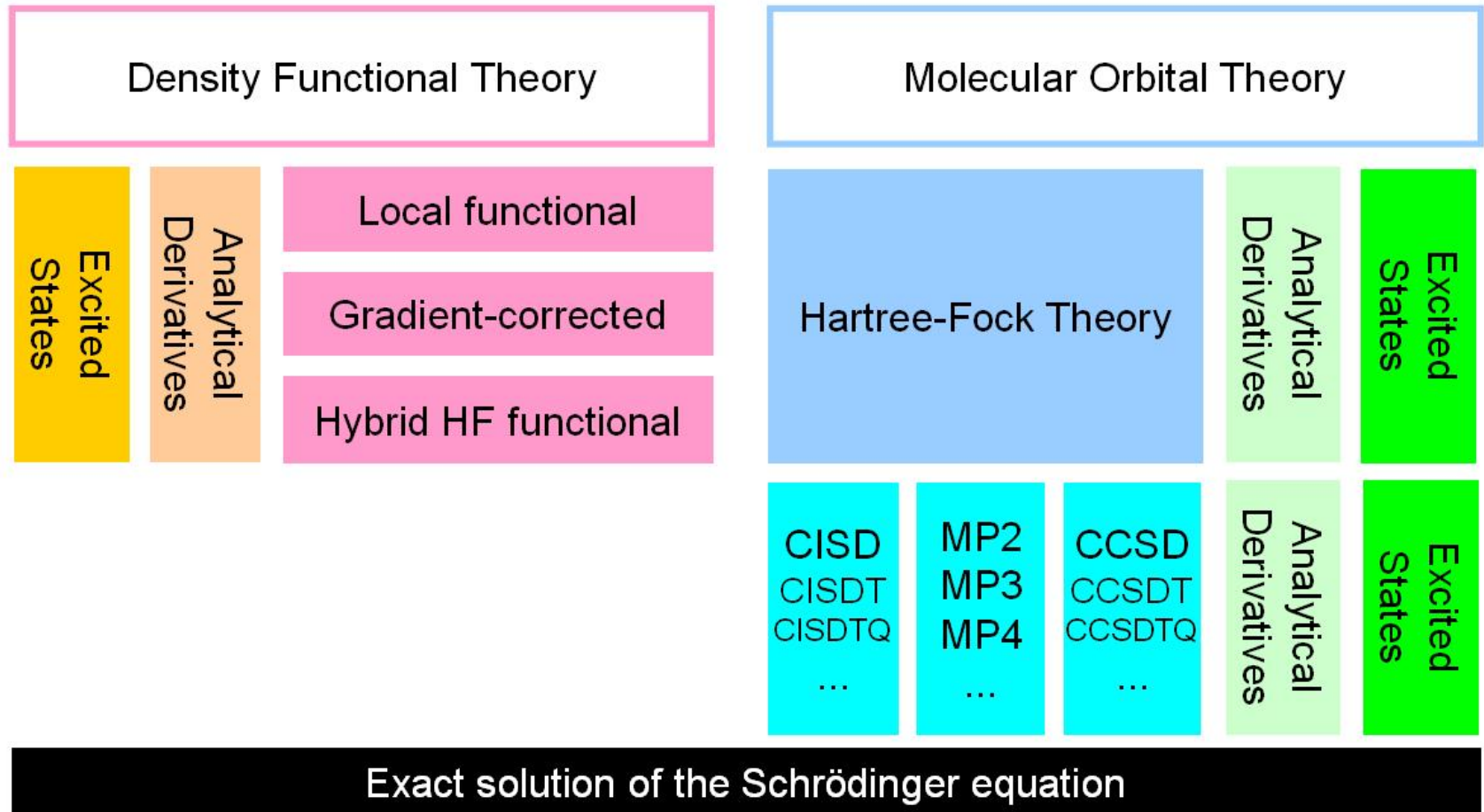
Conditions arising from the indistinguishability of electrons

$$\begin{aligned} &\Psi(\mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e3}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_O) = \\ &-\Psi(\mathbf{r}_{e2}, \mathbf{r}_{e1}, \mathbf{r}_{e3}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_O) = \\ &\Psi(\mathbf{r}_{e3}, \mathbf{r}_{e1}, \mathbf{r}_{e2}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_O) = \\ &-\Psi(\mathbf{r}_{e3}, \mathbf{r}_{e2}, \mathbf{r}_{e1}, \mathbf{r}_{e4}, \mathbf{r}_{e5}, \mathbf{r}_{e6}, \mathbf{r}_{e7}, \mathbf{r}_{e8}, \mathbf{r}_{e9}, \mathbf{r}_{e10}, \mathbf{r}_{H1}, \mathbf{r}_{H2}, \mathbf{r}_O) = \dots \end{aligned}$$

Many-body

3,628,800 terms!

Systematic many-body methods



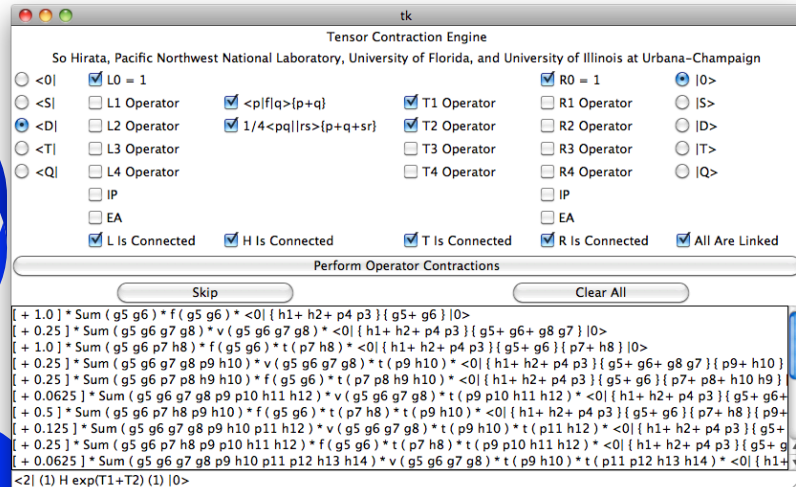
Automated symbolic algebra

Definition of a many-electron theory

$$E = \langle \Phi_0 | e^{-T_1 - T_2} H e^{T_1 + T_2} | \Phi_0 \rangle; \quad 0 = \langle \Phi_i^a | e^{-T_1 - T_2} H e^{T_1 + T_2} | \Phi_0 \rangle; \quad 0 = \langle \Phi_{ij}^{ab} | e^{-T_1 - T_2} H e^{T_1 + T_2} | \Phi_0 \rangle$$

Mathematical expressions

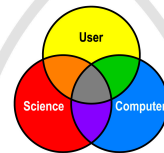
$$\begin{aligned} (\Xi_1)_{h_1 p_0}^{h_7 h_{10}} &= +t_{h_1}^{p_0} v_{p_0 p_6}^{h_7 h_{10}} \\ (\Xi_2)_{h_1 h_2}^{h_{10} h_{11}} &= -\frac{1}{2} t_{h_1}^{p_8} v_{p_8 p_6}^{h_{10} h_{11}} \\ (\Xi_3)_{h_1 p_5}^{h_{10} p_3} &= -\frac{1}{2} t_{h_1}^{p_6} v_{p_5 p_6}^{h_{10} p_3} \\ (\Xi_4)_{p_5}^{h_{10}} &= -t_{h_7}^{p_5} v_{p_5 p_6}^{h_7 h_{10}} \\ (\xi_{22})_{h_1 p_5}^{h_{10} h_{11}} &= +v_{h_1 p_5}^{h_{10} h_{11}} + \frac{1}{2} (\Xi_1)_{h_1 p_5}^{h_{10} h_{11}} \\ (\xi_{22})_{h_1 h_2}^{h_{10} h_{11}} &= -v_{h_1 h_2}^{h_{10} h_{11}} + P_2 v_{h_1}^{p_5} (\xi_{22})_{h_2 p_5}^{h_{10} h_{11}} + (\Xi_2)_{h_1 h_2}^{h_{10} h_{11}} \\ (\xi_{23})_{h_1 p_5}^{h_{10} p_3} &= +v_{h_1 p_5}^{h_{10} p_3} + (\Xi_3)_{h_1 p_5}^{h_{10} p_3} \\ (\xi_{24})_{p_5}^{h_{10}} &= +f_{p_5}^{h_{10}} + (\Xi_4)_{p_5}^{h_{10}} \\ (\xi_{25})_{h_1 p_9}^{h_7 h_{10}} &= +v_{h_1 p_9}^{h_7 h_{10}} + (\Xi_1)_{h_1 p_9}^{h_7 h_{10}} \\ (\xi_2)_{h_1 h_2}^{h_{10} p_3} &= +v_{h_1 h_2}^{h_{10} p_3} + \frac{1}{2} t_{h_1}^{p_3} (\xi_{22})_{h_1 h_2}^{h_{10} h_{11}} - P_2 v_{h_1}^{p_5} (\xi_{23})_{h_2 p_5}^{h_{10} p_3} \\ &\quad - t_{h_1 h_2}^{p_3} (\xi_{24})_{p_5}^{h_{10}} + P_2 t_{h_1 h_7}^{p_3} (\xi_{25})_{h_2 p_9}^{h_7 h_{10}} + \frac{1}{2} t_{h_1 h_2}^{p_6} v_{p_5 p_6}^{h_{10} p_3} \\ (\xi_3)_{h_1 p_5}^{p_3 p_4} &= +f_{h_1 p_5}^{p_3 p_4} - \frac{1}{2} t_{h_1}^{p_6} v_{p_5 p_6}^{p_3 p_4} \\ (\xi_{42})_{p_8}^{h_9} &= +f_{p_8}^{h_9} + (\Xi_4)_{p_8}^{h_9} \\ (\xi_4)_{h_1}^{h_9} &= +f_{h_1}^{h_9} + t_{p_8}^{h_9} (\xi_{42})_{p_8}^{h_9} - t_{h_7}^{p_8} v_{h_1 p_8}^{h_7 h_9} - \frac{1}{2} t_{h_1 h_8}^{p_8} v_{p_8 p_7}^{h_8 h_9} \\ (\xi_5)_{p_5}^{p_3} &= +f_{p_5}^{p_3} - t_{p_5}^{p_3} v_{p_5 p_6}^{h_7 h_8} - \frac{1}{2} t_{h_7 h_8}^{p_3} v_{p_5 p_6}^{h_7 h_8} \\ (\xi_{62})_{h_1 p_8}^{h_9 h_{11}} &= +v_{h_1 p_8}^{h_9 h_{11}} + \frac{1}{2} (\Xi_1)_{h_1 p_8}^{h_9 h_{11}} \\ (\xi_6)_{h_1 h_2}^{h_9 h_{11}} &= -v_{h_1 h_2}^{h_9 h_{11}} + P_2 v_{h_1}^{p_3} (\xi_{62})_{h_2 p_3}^{h_9 h_{11}} + (\Xi_2)_{h_1 h_2}^{h_9 h_{11}} \\ (\xi_7)_{h_1 p_5}^{h_9 p_3} &= +v_{h_1 p_5}^{h_9 p_3} + 2 (\Xi_3)_{h_1 p_5}^{h_9 p_3} - \frac{1}{2} t_{h_1 h_8}^{p_3} v_{p_5 p_6}^{h_9 h_{11}} \\ r_{p_3 p_4}^{h_1 h_2} &= +v_{h_1 h_2}^{p_3 p_4} - P_2 t_{h_1}^{p_3} (\xi_2)_{h_1 h_2}^{h_{10} p_3} - P_2 t_{h_1}^{p_5} (\xi_3)_{h_2 p_5}^{p_3 p_4} - P_2 t_{h_1 h_9}^{p_3} (\xi_4)_{h_2}^{h_9} + P_2 t_{h_1 h_2}^{p_3} (\xi_5)_{p_4}^{h_9} \\ &\quad - \frac{1}{2} t_{h_9 h_{11}}^{p_3} (\xi_6)_{h_1 h_2}^{h_9 h_{11}} - P_4 v_{h_1 h_6}^{p_3} (\xi_7)_{h_2 p_5}^{h_9 p_3} + \frac{1}{2} t_{h_1 h_2}^{p_6} v_{p_5 p_6}^{p_3 p_4} \end{aligned}$$



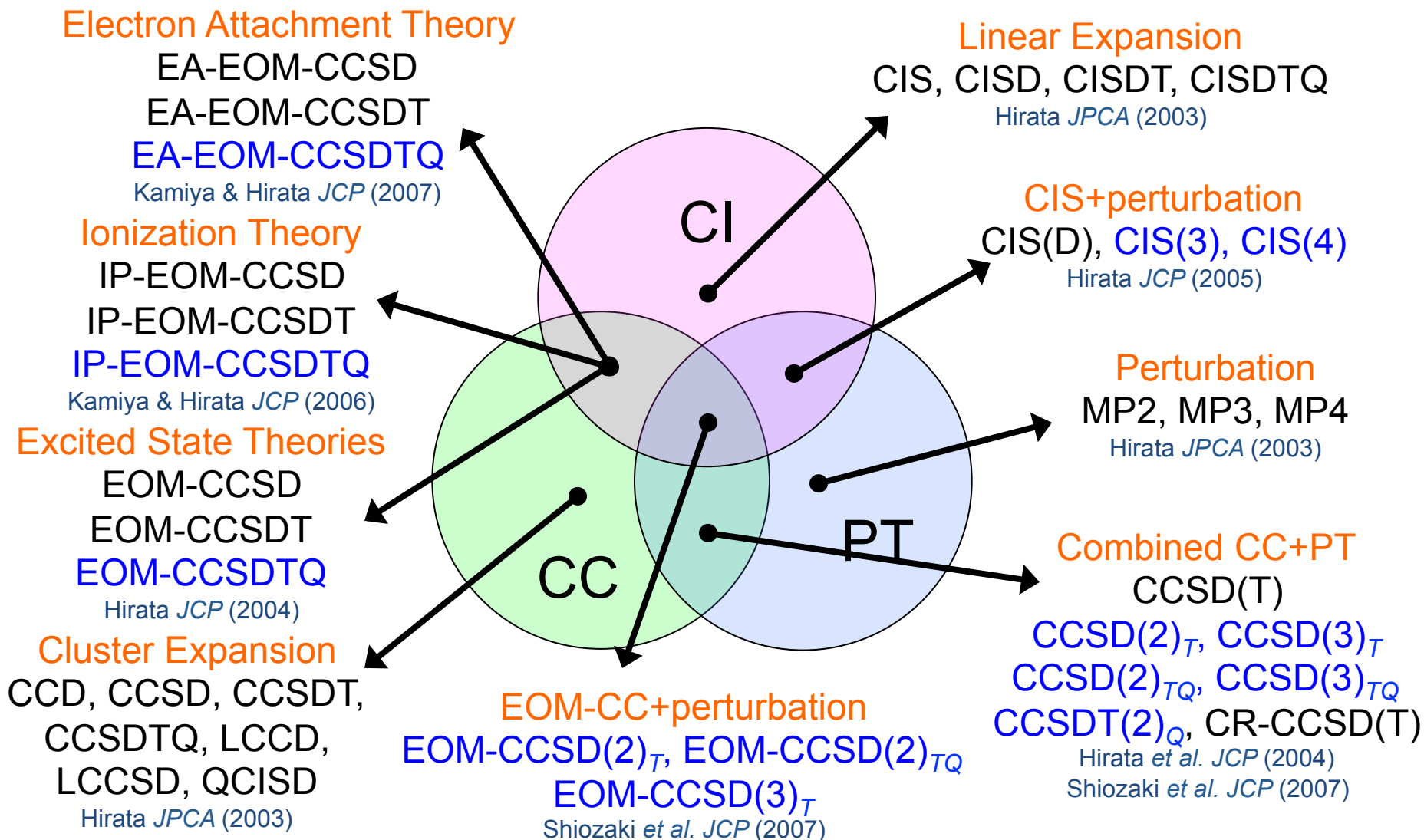
A parallel computer program

```
nprocs = GA_NNODES()
count = 0
next = NXTUAL(nprocs)
DO p3b = noab+1, noab+nvab
DO h6b = 1, noab
DO h1b = 1, noab
DO p5b = noab+1, noab+nvab
IF (next.eq.count) THEN
  IF (.not.restricted).or.(int_mb(k_spin+p
&)+int_mb(k_spin+h1b-1)+int_mb(k_spin+p5b-
  IF (int_mb(k_spin+p3b-1)+int_mb(k_spin+h6
&1b-1)+int_mb(k_spin+p5b-1)) THEN
  IF (ieor(int_mb(k_sym+p3b-1),ieor(int_mb(
&k_sym+h1b-1),int_mb(k_sym+p5b-1)))) .eq.
  &EN
  DO p7b = noab+1, noab+nvab
  DO h8b = 1, noab
  IF (int_mb(k_spin+p3b-1)+int_mb(k_spin+p7
&1b-1)+int_mb(k_spin+h8b-1)) THEN
  IF (ieor(int_mb(k_sym+p3b-1),ieor(int_mb(
&k_sym+h1b-1),int_mb(k_sym+h8b-1)))) .eq.
  IF ((restricted).and.(int_mb(k_spin+p3b-1
&t_mb(k_spin+h1b-1)+int_mb(k_spin+h8b-1)).e
```

Interface Between the User and the Software

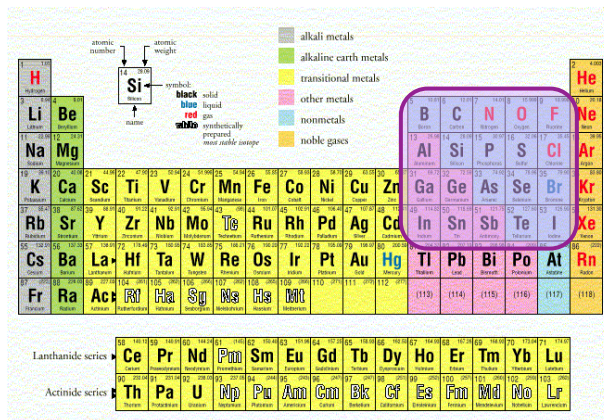
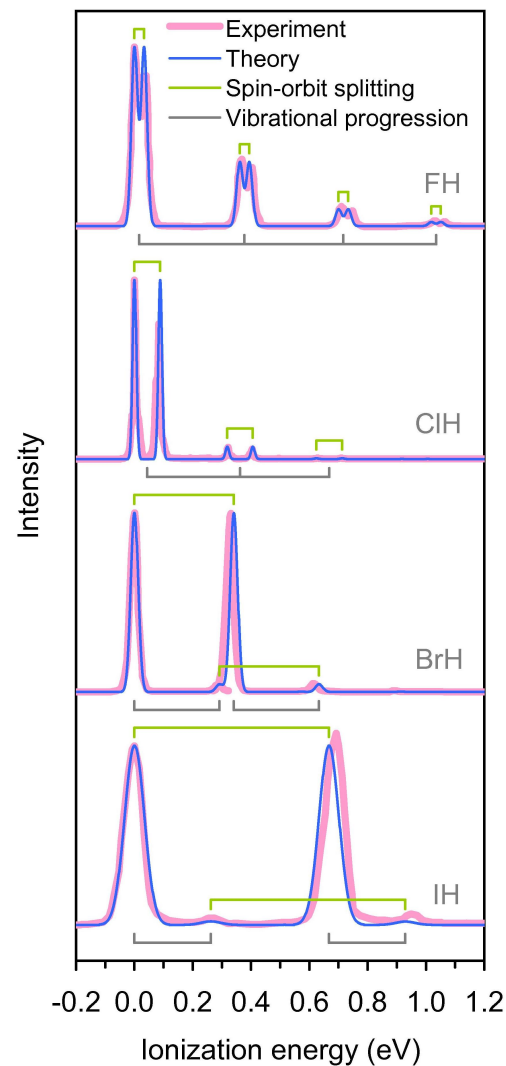
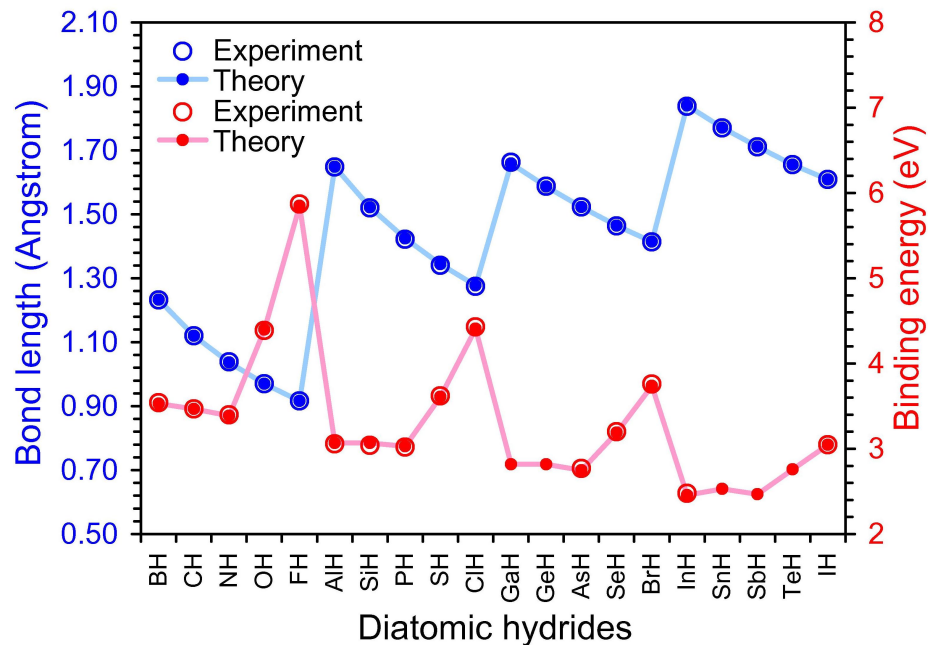


Implemented methods

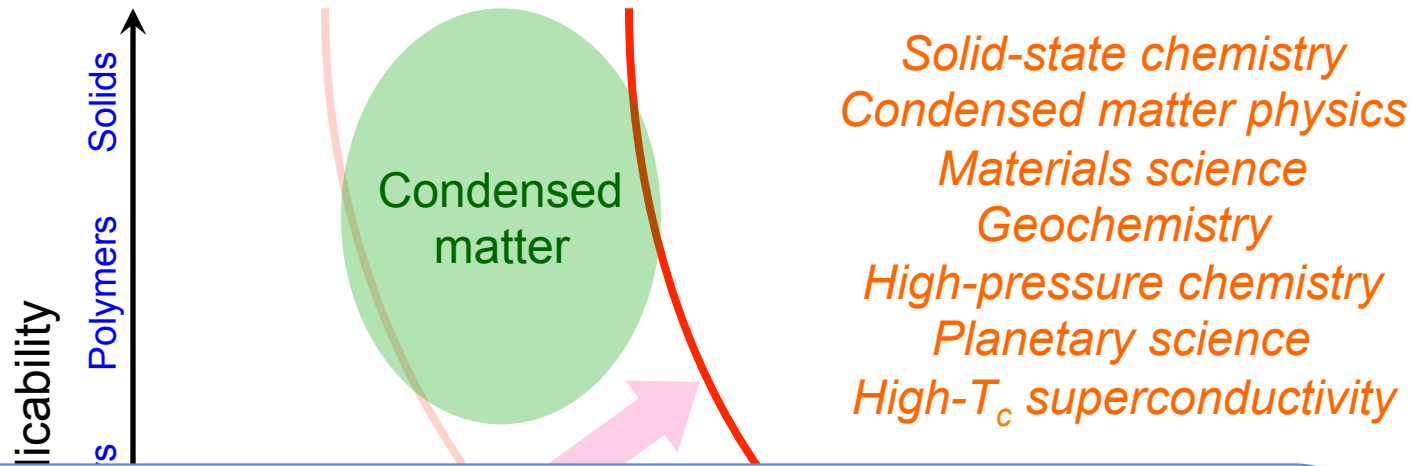


Structures, thermochemistry, and spectra

Hirata et al., *J. Chem. Phys.* (2004); Hirata et al., *J. Chem. Phys.* (2007)



Frontiers of predictive computing



A DOE report on Computational Materials Science (2010):

“We are at the threshold of a new era where the integrated synthesis, characterization, and modeling of complex materials and chemical processes will transform our ability to understand and design new materials and chemistries with predictive power”

Materials Genome Initiative for Global Competitiveness (2011):

“the development of advanced materials can be accelerated through advances in computational techniques”



SciDAC

Scientific Discovery
through
Advanced Computing

So Hirata (Chemistry, UIUC) - Lead PI

Hirata is a theoretical/computational chemist and an expert in electron-correlation theories for molecules and solids. He is the primary author of the computer-generated, high-rank electron-correlation modules in DOE's massively parallel NWCHEM suite of software, implementing several of his original methods.



Lucas Wagner (Physics, UIUC) - Co-lead PI

Wagner is the principal author of the quantum Monte Carlo program, QWALK, with which he has performed predictively accurate calculations on strongly correlated systems.



Peter Abbamonte (Physics, UIUC)

Abbamonte, an experimental condensed-matter physicist, brings an invaluable experimental insight into the project. He is one of the originators of resonant soft x-ray scattering, with which he discovered a Wigner crystal in doped spin ladders and the charged stripes in copper-oxide superconductors.



David Ceperley (Physics & NCSA, UIUC)

Ceperley is a theoretical/computational physicist and an authority of quantum Monte Carlo (QMC). He invented a number of QMC algorithms and is the author of massively parallel QMCPACK software.



Garnet K.-L. Chan (Chemistry, Princeton)

Chan is a recognized expert in strong correlation theories including density matrix renormalization group (DMRG), tensor networks, and density matrix functional theory. He has, in particular, established DMRG as a practical, powerful tool for strongly correlated molecular electronic structures.



Bryan Clark (Microsoft Station Q)

Clark has considerable experience in both conventional quantum Monte Carlo (QMC) and novel extensions such as QMC in the Hilbert space. He has developed a large-scale parallel algorithm of QMC in PIMC++.



Shinsei Ryu (Physics, UIUC)

Ryu specializes in mathematical theories of strong correlation and other condensed phase electronic structures. Ryu is a pioneer in the use of entanglement entropy in classifying topological phases of matter and has predicted the fractional topological insulator in two dimension.



Shiwei Zhang (Physics, W&M)

Zhang specializes in computational condensed matter physics and materials science. Zhang is a pioneer in the use of quantum Monte Carlo (QMC) in the Hilbert space for strong correlation. He is the inventor of the phaseless auxiliary field QMC.

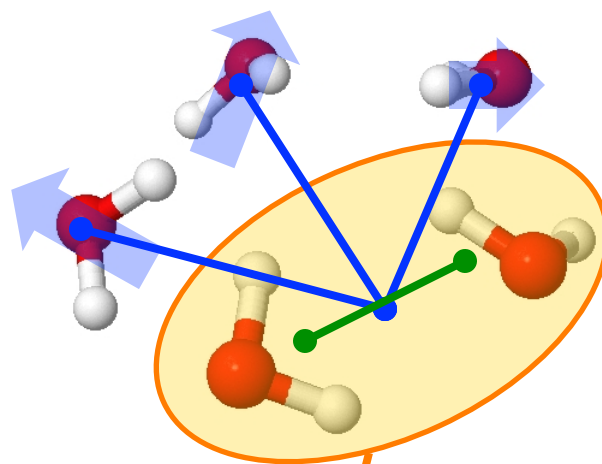


Embedded-fragment approach

Hirata *et al.*, *Mol. Phys.* (2005); Kamiya, Hirata, and Valiev, *J. Chem. Phys.* (2008);
Hirata *et al.*, *Acc. Chem. Res.* (2014)

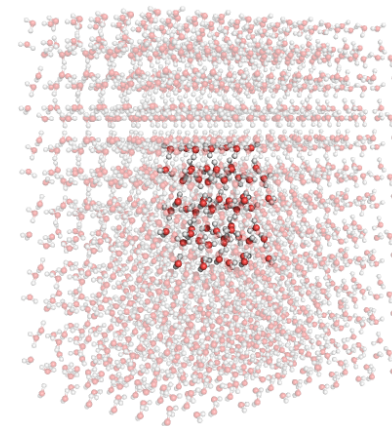
N -body ($N > 2$) Coulomb in point-charge or dipole approximation

1 and 2-body
Coulomb
Exchange
Correlation



Pair energy in the
presence of self-
consistent atomic
charges or dipoles

$$E = \sum_{i=1}^n E'_i + \sum_{i<j}^n \left(E'_{ij} - E'_i - E'_j \right) + \dots$$



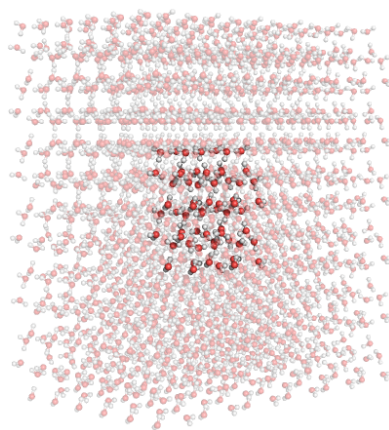
Cf. Li, Paulus, Schutz, Manby, Beran, Truhlar, Raghavachari, Herbert,
Collins, Gordon, Gao, Fedorov, Kitaura, Zhang, et al.

Ice Ih

He, Sode, Xantheas, and Hirata, *J. Chem. Phys.* (2012)



Xiao He



MP2/aug-cc-pVDZ

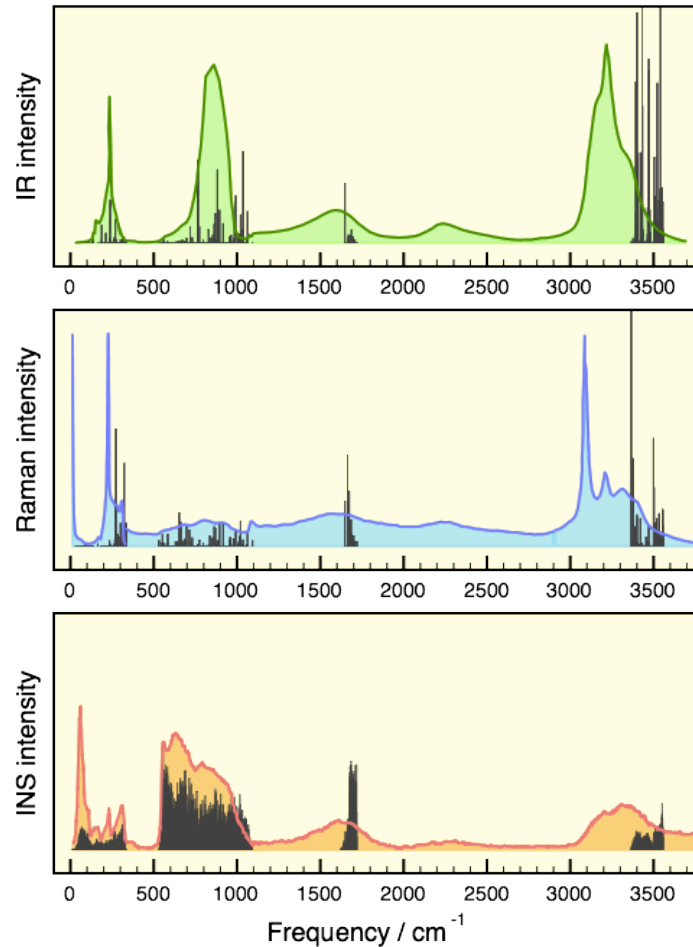
Ice Ih

He, Sode, Xantheas, and Hirata, *J. Chem. Phys.* (2012)



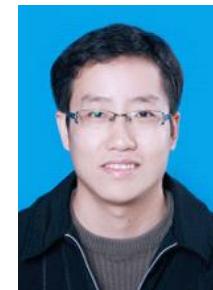
Xiao He

IR, Raman, and Inelastic neutron scattering spectra



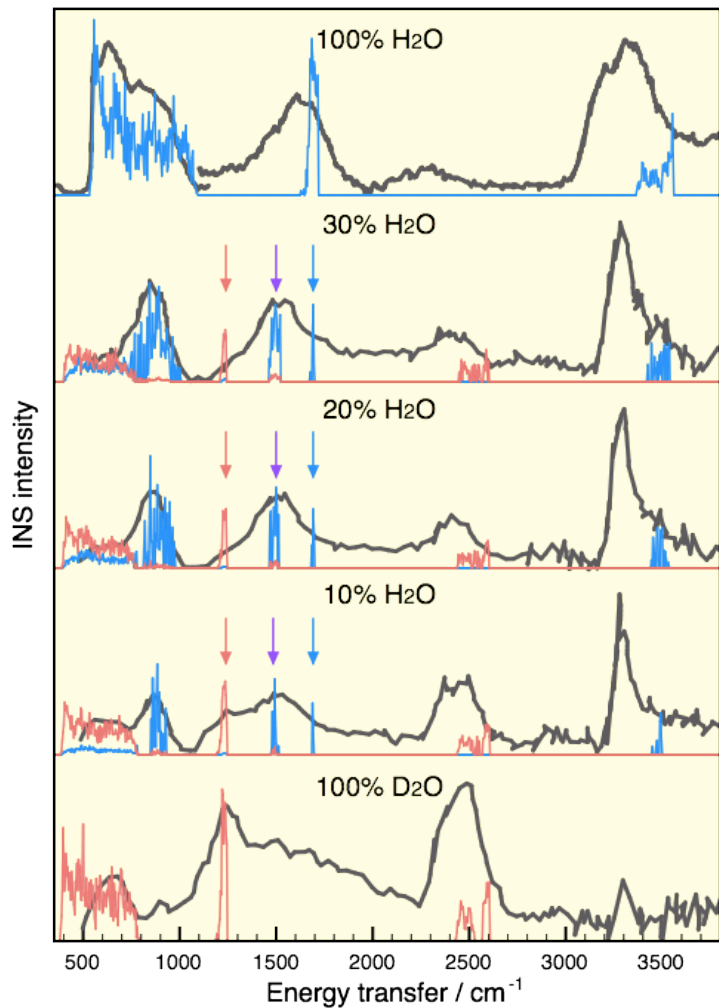
Ice Ih

He, Sode, Xantheas, and Hirata, *J. Chem. Phys.* (2012)

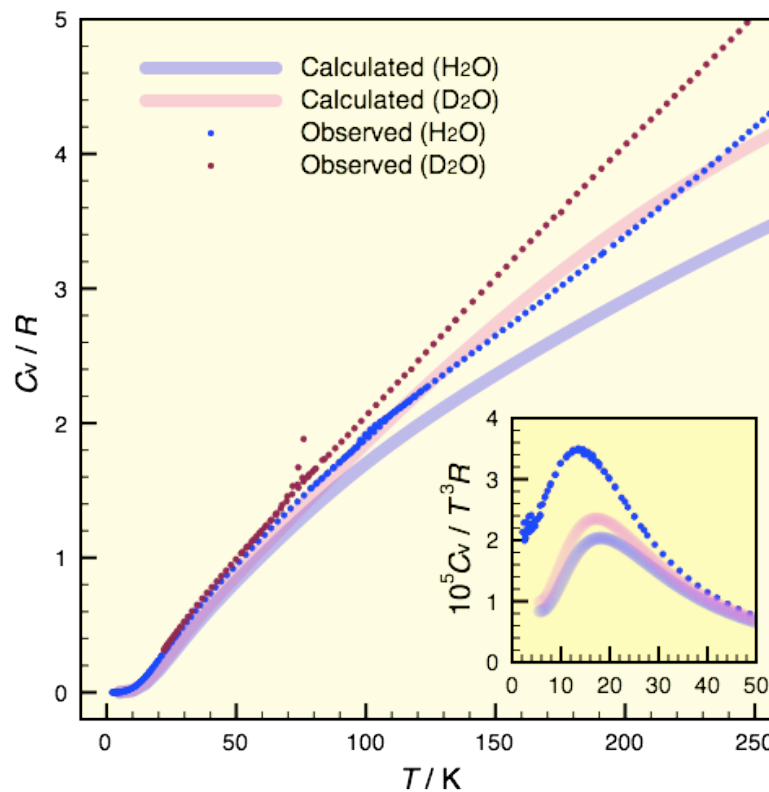


Xiao He

Inelastic neutron scattering spectra



Heat capacities

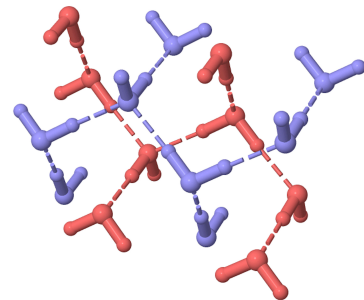
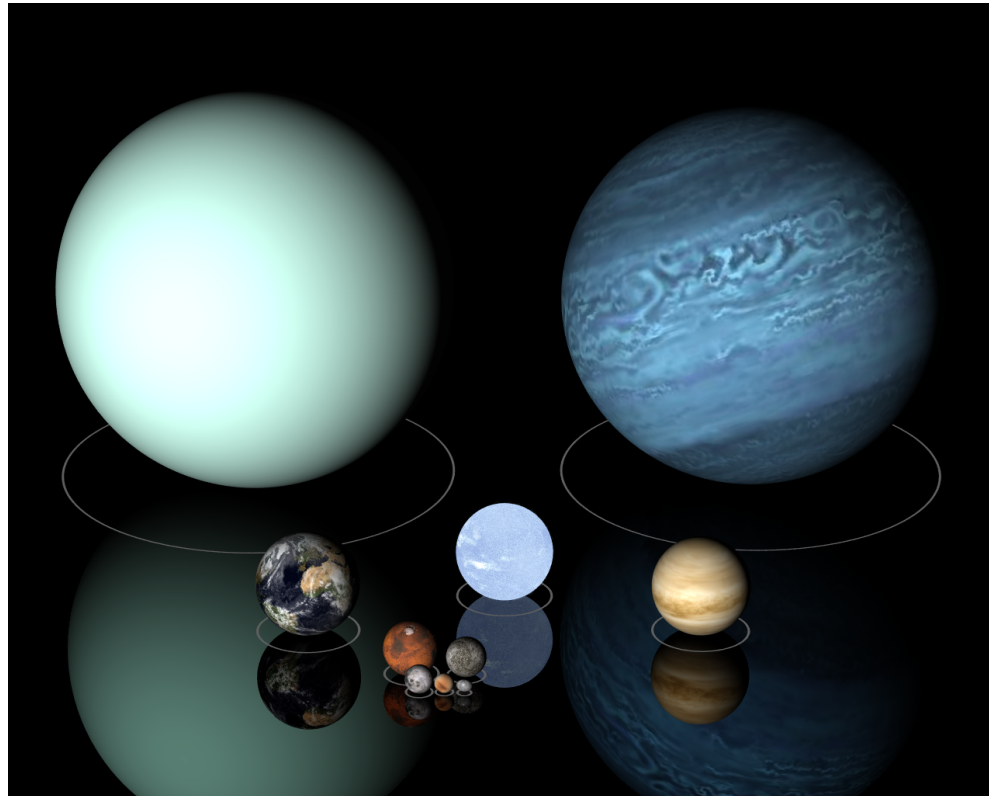


Ice VIII

Gilliard, Sode, and Hirata, *J. Chem. Phys.* (2014)



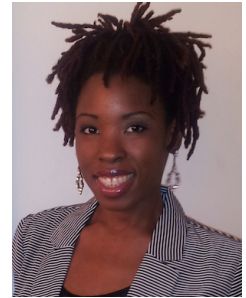
Kandis Gilliard



MP2/aug-cc-pVDZ
CCSD/aug-cc-pVDZ

Ice VIII

Gilliard, Sode, and Hirata, *J. Chem. Phys.* (2014)



Kandis Gilliard

The New York Times

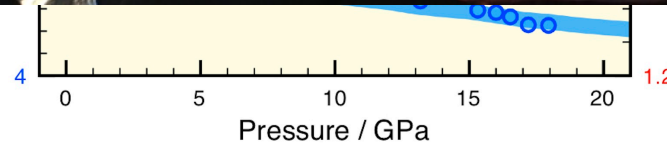
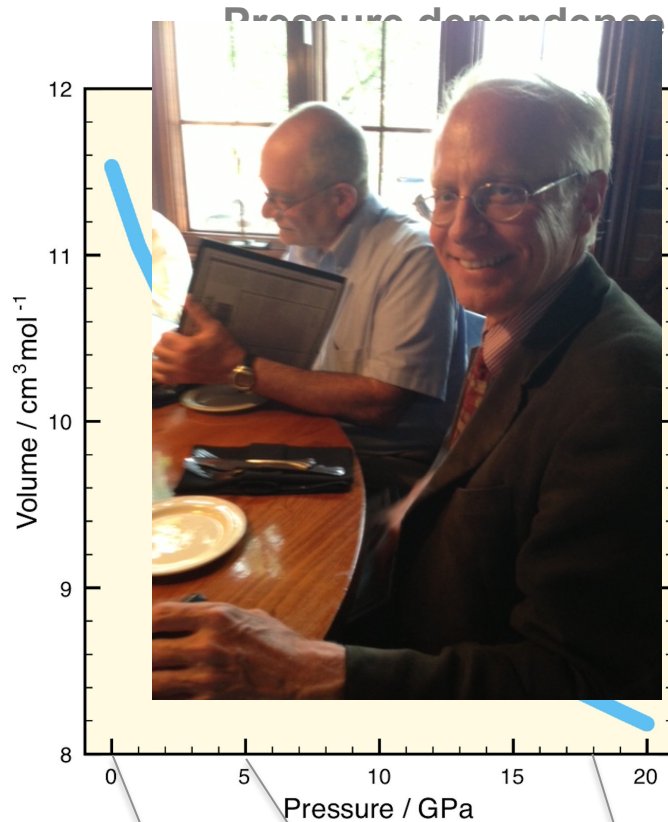
December 16, 2013

The Big Squeeze

By KENNETH CHANG

WASHINGTON — In a recurring comic bit, David Letterman used to place household items — a plate of jelly doughnuts, a six-pack of beer — in an 80-ton hydraulic press, gleefully watching as the items squirmed, exploded and disintegrated.

That was but a light touch compared with the pressures Russell J. Hemley and his colleagues exert on molecules at the Carnegie Institution for Science here.



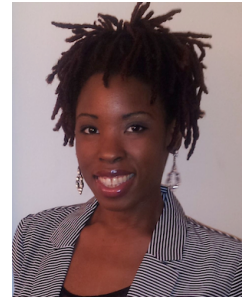
Diamond synthesis (18 GPa)

Center of the Moon (5 GPa)

Bottom of the Mariana Trench (0.1 GPa)

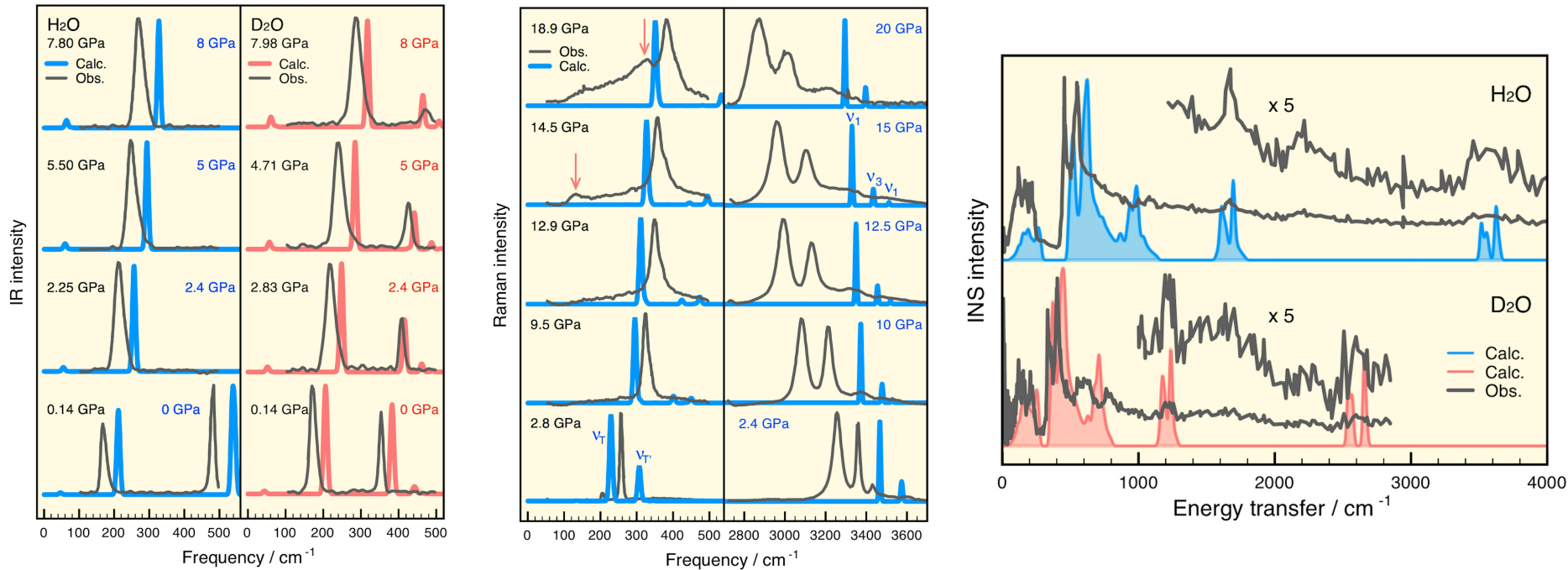
Ice VIII

Gilliard, Sode, and Hirata, *J. Chem. Phys.* (2014)



Kandis Gilliard

Pressure dependence of IR, Raman, and INS spectra

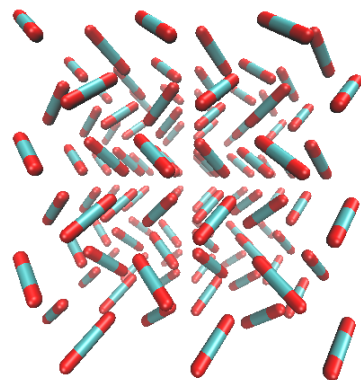


Solid CO₂

Sode, Keçeli, Yagi, and Hirata, *J. Chem. Phys.* (2012)



Olaseni Sode



MP2/aug-cc-pVDZ

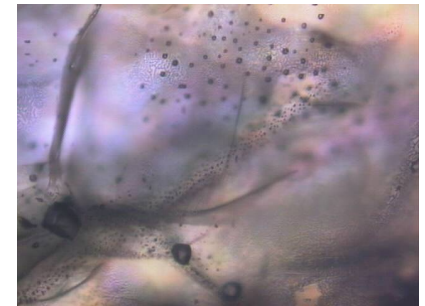
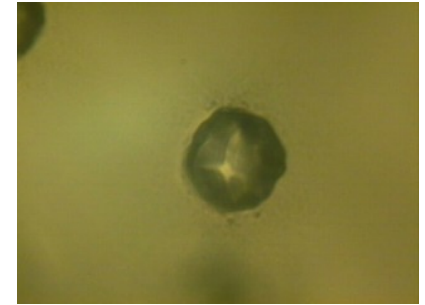
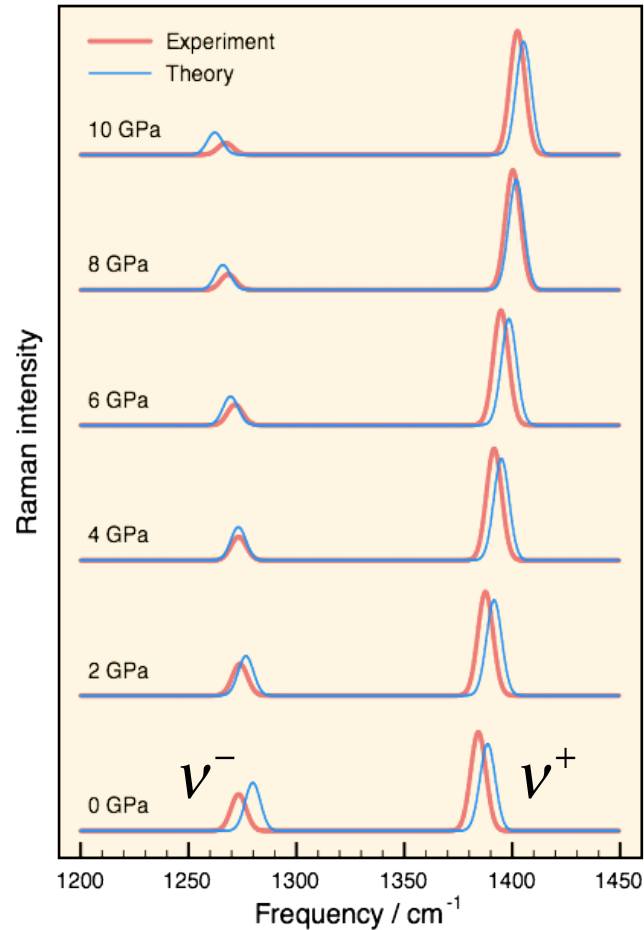
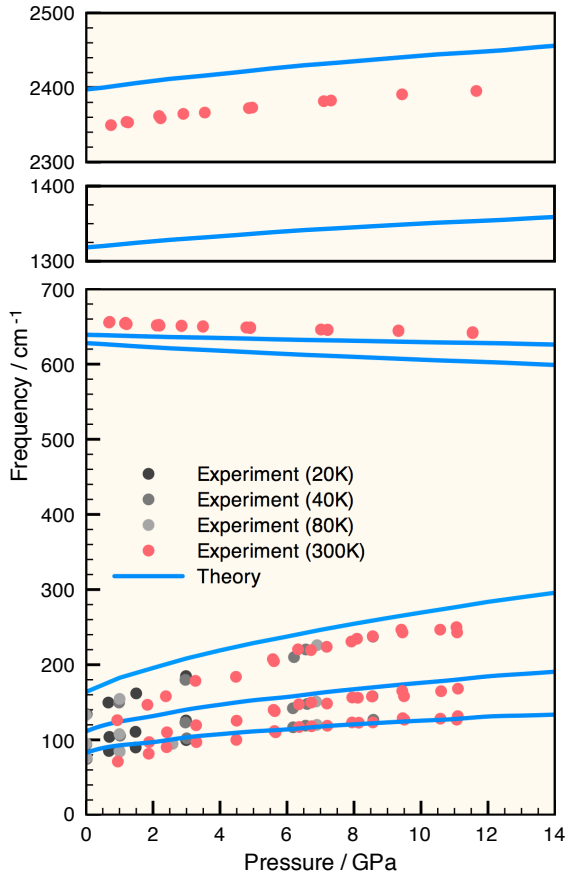
Solid CO₂

Sode, Keçeli, Yagi, and Hirata, *J. Chem. Phys.* (2012)



Olaseni Sode

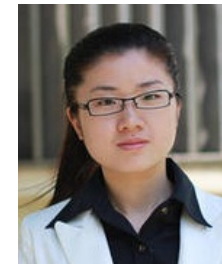
Pressure dependence of Raman spectra



CO₂ encapsulation
in minerals

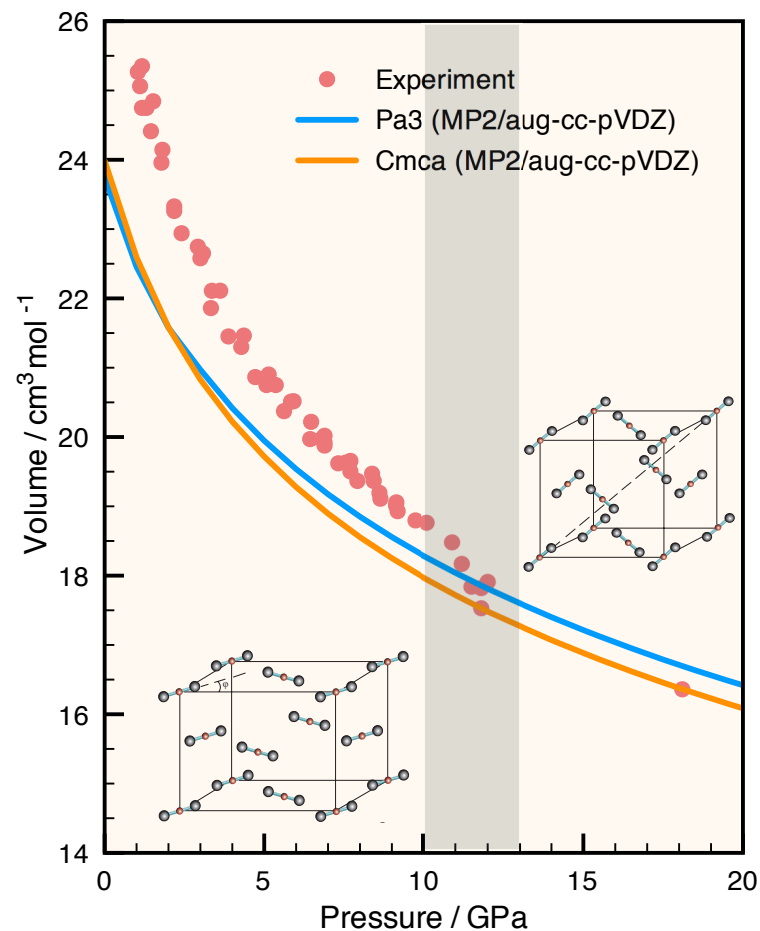
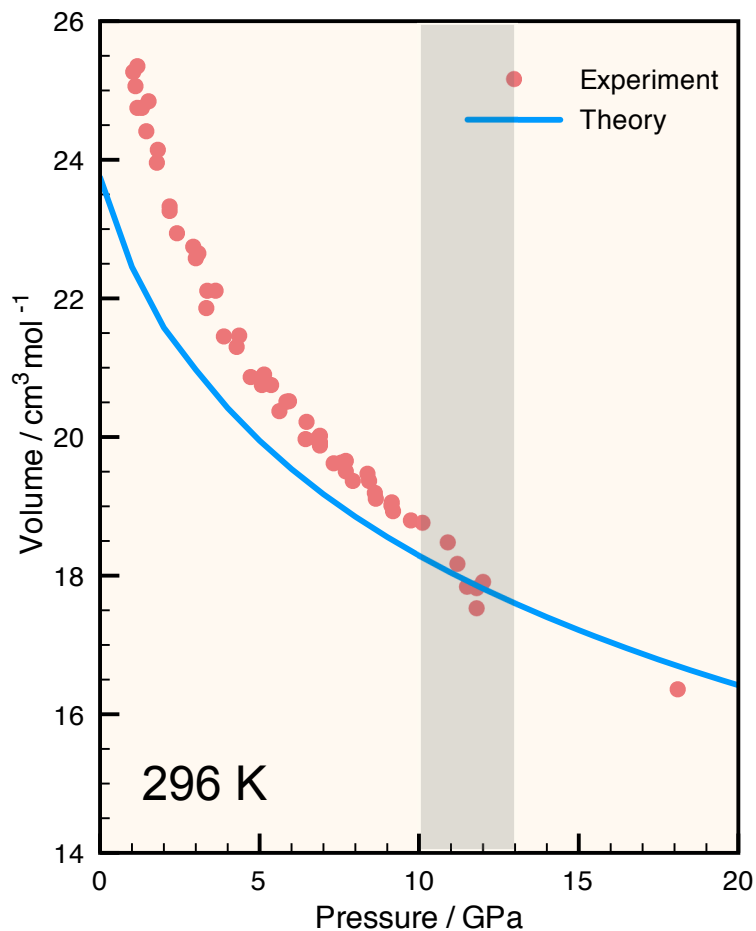
Phase transition in solid CO₂

Li, Sode, Voth and Hirata, *Nature Communications* (2013)



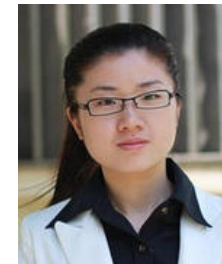
Jinjin Li

Pressure dependence of volume

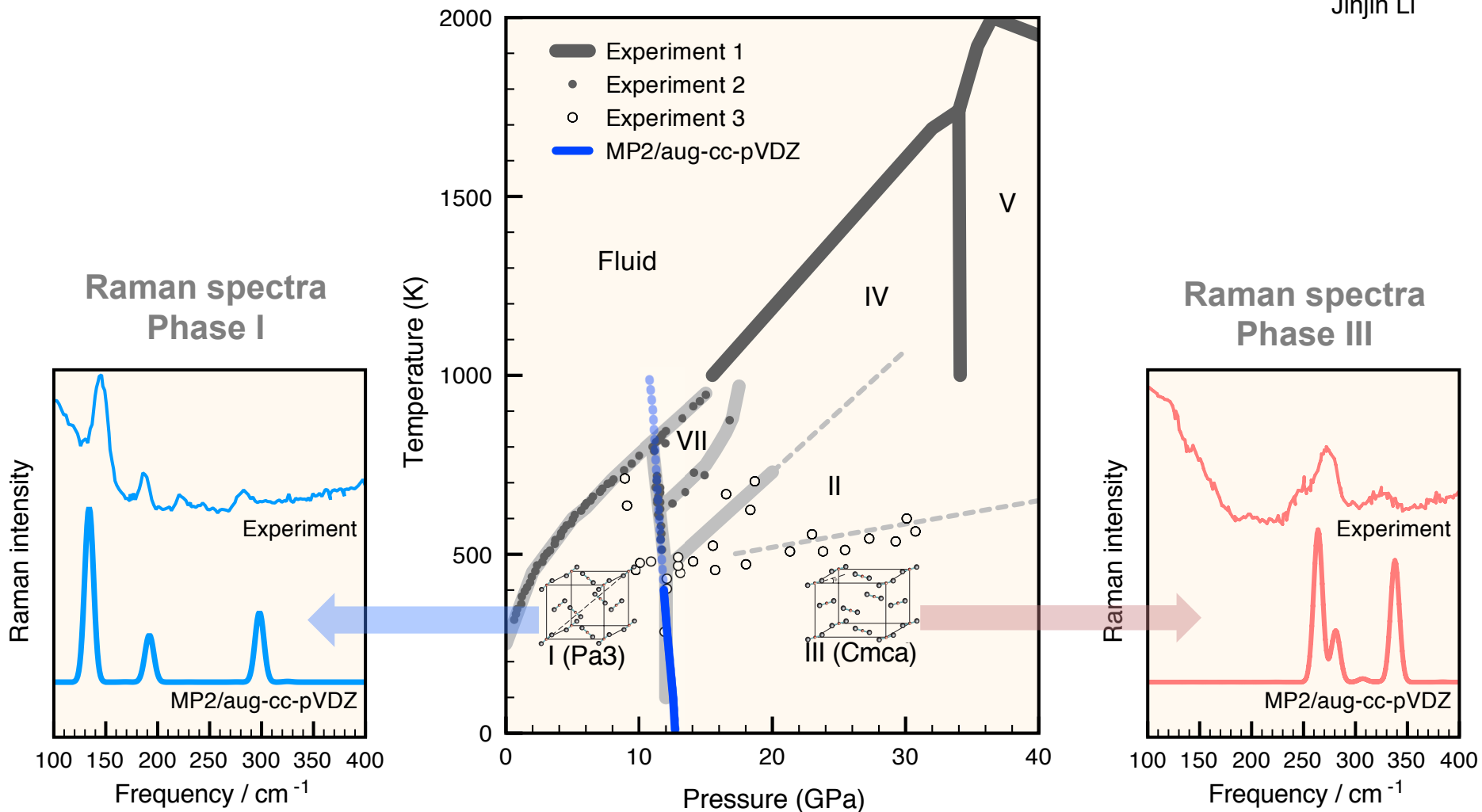


Phase transition in solid CO₂

Li, Sode, Voth and Hirata, *Nature Communications* (2013)

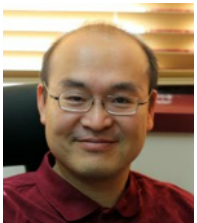


Jinjin Li

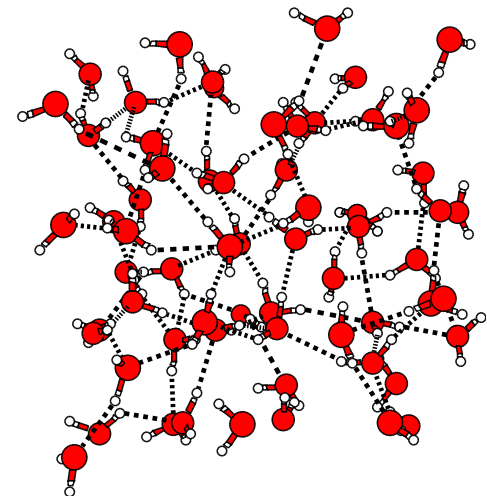
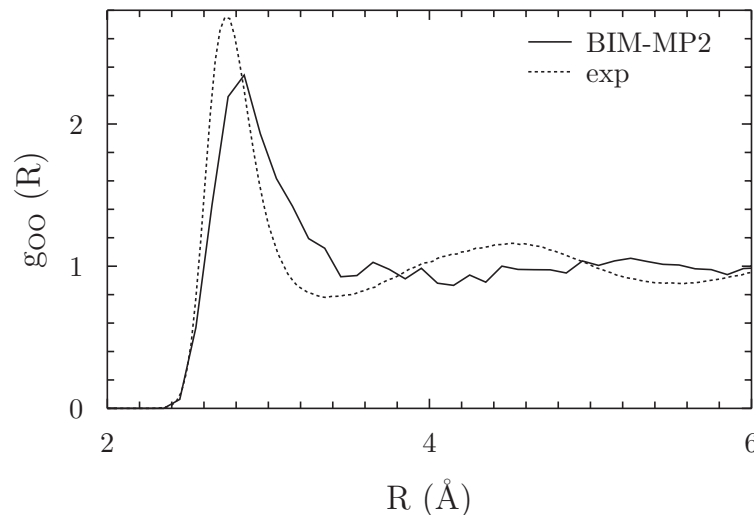


What can be done with Blue Waters?

- First-principles phase diagram of ice
- First-principles prediction of thermal expansion of ice
- First-principles simulation of liquid water
- First-principles simulation of chemical reactions in aqueous media



Soohaeng Willow





Monte Carlo MP2

Willow, Kim and Hirata, *J. Chem. Phys.* (2012)

Very long $O(n^4)$ summation of products of two 6-dimensional integrals

$$E^{(2)} = \sum_{i,j}^{\text{occ.}} \sum_{a,b}^{\text{vir.}} \frac{\langle ab|ij\rangle \langle ij|ab\rangle}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$

Explicit two-electron integrals

$$E^{(2)} = \sum_{i,j}^{\text{occ.}} \sum_{a,b}^{\text{vir.}} \frac{\int \varphi_i(\mathbf{r}_1) \varphi_j(\mathbf{r}_2) \frac{1}{r_{12}} \varphi_a(\mathbf{r}_1) \varphi_b(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \int \varphi_i(\mathbf{r}_3) \varphi_j(\mathbf{r}_4) \frac{1}{r_{34}} \varphi_a(\mathbf{r}_3) \varphi_b(\mathbf{r}_4) d\mathbf{r}_3 d\mathbf{r}_4}{\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b}$$

Laplace transformation of the denominator

$$E^{(2)} = - \sum_{i,j}^{\text{occ.}} \sum_{a,b}^{\text{vir.}} \int \varphi_i(\mathbf{r}_1) \varphi_j(\mathbf{r}_2) \frac{1}{r_{12}} \varphi_a(\mathbf{r}_1) \varphi_b(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \int \varphi_i(\mathbf{r}_3) \varphi_j(\mathbf{r}_4) \frac{1}{r_{34}} \varphi_a(\mathbf{r}_3) \varphi_b(\mathbf{r}_4) d\mathbf{r}_3 d\mathbf{r}_4 \int_0^\infty e^{(\varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b)\tau} d\tau$$

Change of orders of summations and integrations

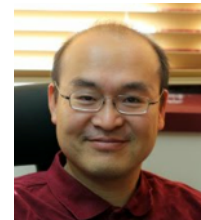
$$E^{(2)} = - \int \dots \int \int_0^\infty \frac{\sum_i^{\text{occ.}} \varphi_i(\mathbf{r}_1) \varphi_i(\mathbf{r}_3) e^{\varepsilon_i \tau} \sum_j^{\text{occ.}} \varphi_j(\mathbf{r}_2) \varphi_j(\mathbf{r}_4) e^{\varepsilon_j \tau} \sum_a^{\text{vir.}} \varphi_a(\mathbf{r}_1) \varphi_a(\mathbf{r}_3) e^{-\varepsilon_a \tau} \sum_b^{\text{vir.}} \varphi_b(\mathbf{r}_2) \varphi_b(\mathbf{r}_4) e^{-\varepsilon_b \tau}}{r_{12} r_{34}} d\mathbf{r}_1 \dots d\mathbf{r}_4 d\tau$$

Single 13-dimensional integral evaluated by Monte Carlo

$$E^{(2)} = - \int \dots \int \int_0^\infty \frac{G^-(\mathbf{r}_1, \mathbf{r}_3, \tau) G^-(\mathbf{r}_2, \mathbf{r}_4, \tau) G^+(\mathbf{r}_1, \mathbf{r}_3, \tau) G^+(\mathbf{r}_2, \mathbf{r}_4, \tau)}{r_{12} r_{34}} d\mathbf{r}_1 \dots d\mathbf{r}_4 d\tau$$

Monte Carlo MP2

Willow, Kim and Hirata, *J. Chem. Phys.* (2012)



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$$E = \int f(x) dx = \int \frac{f(x)}{g(x)} g(x) dx = \sum_{x \in g} \frac{f(x)}{g(x)}$$

g(x) **weight function**

Metropolis

Requirement 1: analytically integrable

$$\int g(x) dx$$

Requirement 2: cancellation of singularities

$$f(x)$$

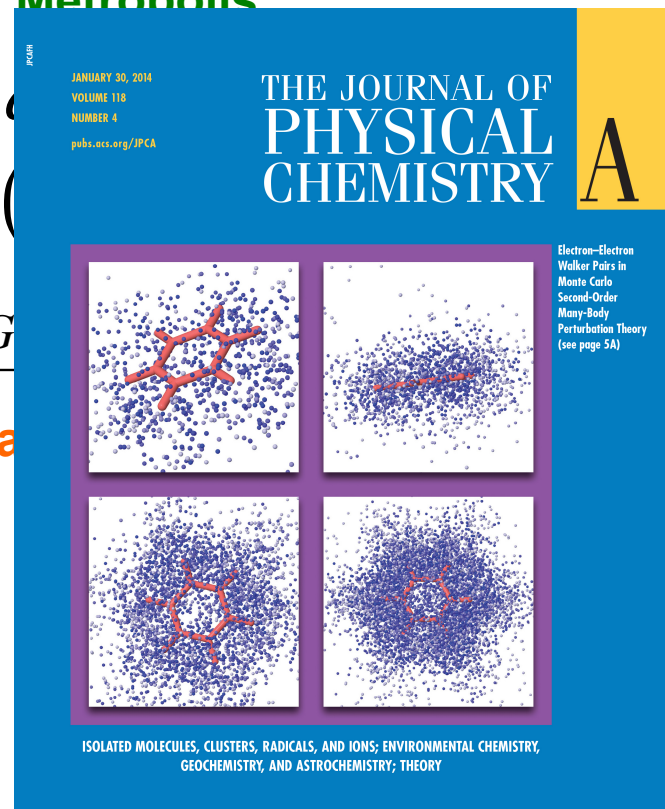
$$E^{(2)} = - \int \dots \int \int_0^\infty \frac{G^-(\mathbf{r}_1, \mathbf{r}_3, \tau) G^-(\mathbf{r}_2, \mathbf{r}_4, \tau) G^+(\mathbf{r}_1, \mathbf{r}_3, \tau) G^+(\mathbf{r}_2, \mathbf{r}_4, \tau)}{r_{12} r_{34}} d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 d\tau$$

$$r_{12} r_{34}$$

Singularities

$$g(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \frac{1}{4E_{\text{Coulomb}}^2} \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{r_{12}} \frac{\rho(\mathbf{r}_3)\rho(\mathbf{r}_4)}{r_{34}}$$

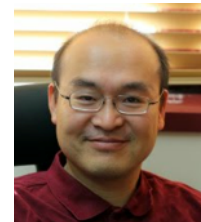
$$\int g(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 = 1$$



MC-MP2 for E_{corr} , IP, and EA

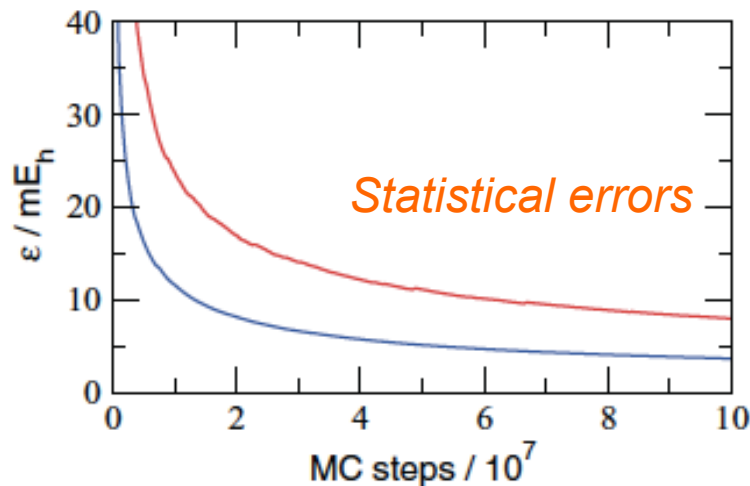
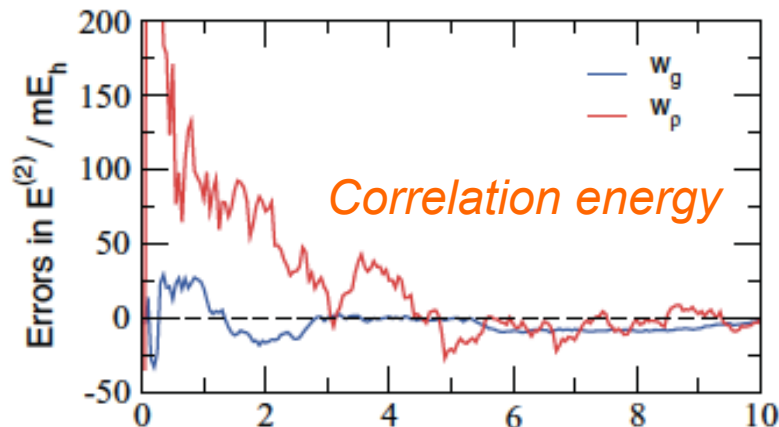
Willow, Kim and Hirata, *J. Chem. Phys.* (2012)

Willow, Kim and Hirata, *J. Chem. Phys.* (2013)

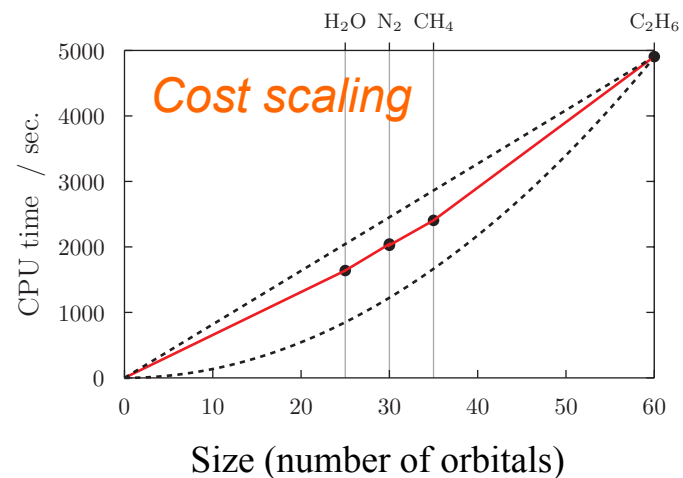
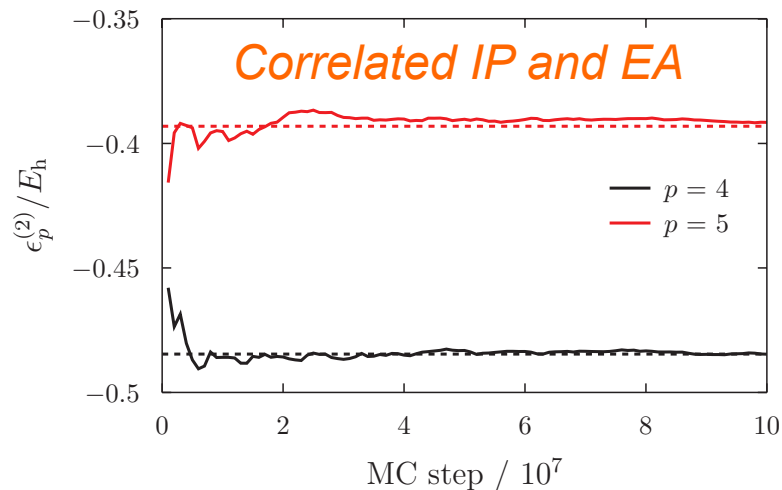


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Nitrogen 6-31G**



Water HOMO and HOMO-1



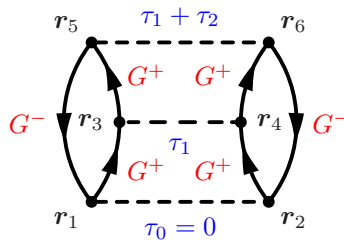
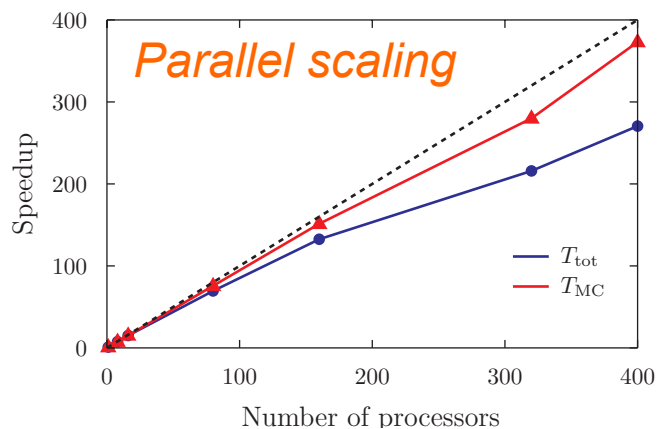
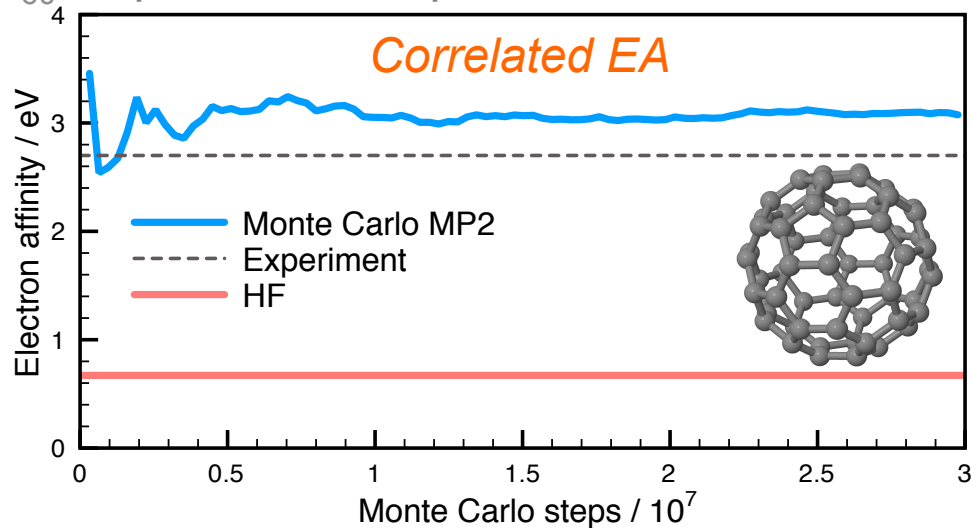
Parallel MC-MP2, MP3, MP2-R12

Willow, Hermes, Kim and Hirata, *J. Chem. Theo. Comput.* (2013); Willow and Hirata, *J. Chem. Phys.* (2013); Willow, Zhang, Valeev, and Hirata, *J. Chem. Phys. (Comm.)* (2014)

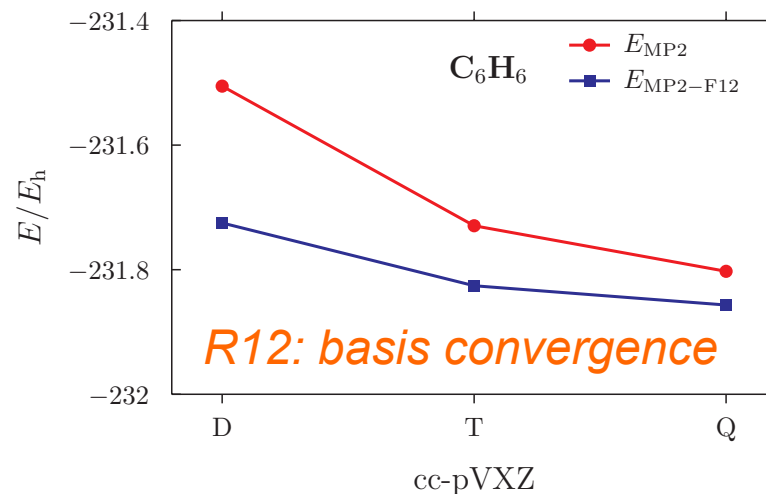


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C_{60} cc-pVDZ on 320 processors of Blue Waters



Diagrammatics



What can be done with Blue Waters?

- Accurate calculations of opto-electronic properties of conjugated polymers used in organic solar cells, LED, FET, capacitors, etc.
- Accurate calculations of van der Waals interactions between conjugated polymers, PAHs, graphene, graphite, C₆₀, etc.



Matthew Hermes



Val. band edge	PPP (1)	PT (2)
Experiment	5.2-5.65 eV	5.3-5.55 eV
DFT	5.0 eV	4.5 eV
HF	6.6 eV	6.2 eV
MP2	6.4 eV	5.5 eV

